$$\lim_{x\to 0} \frac{f(x)-2x}{x^2} = 4$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{f(x)-2x}{x^2}\cdot x^2 + 2x\right) = 4\cdot 0 + 0 = 0$$

$$\Rightarrow \lim_{x\to 0} f(x) = 0$$

$$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0}\left[\frac{\frac{f(x)}{x}-2}{x}\cdot x + 2\right]$$

$$= \lim_{x\to 0}\left[\frac{f(x)-2x}{x^2}\cdot x + 2\right] = 4\cdot 0 + 2$$

$$\Rightarrow \lim_{x\to 0} \frac{f(x)}{x} = 2$$

$$\lim_{x\to 0} \frac{e^{f(x)}-2x-1}{x^2} = \lim_{x\to 0} \frac{e^{f(x)}-f(x)+f(x)-2x-1}{x^2}$$

$$= \lim_{x\to 0}\left[\frac{e^{f(x)}-f(x)-1}{x^2} + \frac{f(x)-2x}{x^2}\right] \quad (I)$$

$$\lim_{x\to 0} \frac{e^{f(x)}-f(x)-1}{x^2} = \lim_{x\to 0}\left[\frac{e^{f(x)}-f(x)-1}{f^2(x)}\cdot\frac{f^2(x)}{x^2}\right] \quad (II)$$

$$\lim_{x\to 0} \frac{f^2(x)}{x^2} = \lim_{x\to 0}\left(\frac{f(x)}{x}\right)^2 \quad \exists \lim_{x\to 0}\frac{f(x)}{x} \quad \left(\lim_{x\to 0}\frac{f(x)}{x}\right)^2$$

$$= 4$$

$$\lim_{x\to 0} \frac{e^{f(x)} - f(x) - 1}{f^2(x)} \quad \overline{f(x) = t} \quad \lim_{t\to 0} \frac{e^t - t - 1}{t^2} \quad \overline{\frac{0}{0}\ \text{DLH}}$$

$$\lim_{t\to 0} \frac{e^t - 1}{2t} = \frac{1}{2} \cdot \frac{de^t}{dt}\bigg|_{t=0} = \frac{1}{2}$$

(II) $\Rightarrow$ $$\lim_{x\to 0} \left[\frac{e^{f(x)} - f(x) - 1}{x^2} \right] = \frac{1}{2} \cdot 4 = 2$$

(I) $\Rightarrow$ $$\lim_{x\to 0} \frac{e^{f(x)} - 2x - 1}{x^2} = 2 + 4 = 6$$